## GPU CONFERENCE

## FAST FIXED-RADIUS NEAREST NEIGHBORS: INTERACTIVE MILLION-PARTICLE FLUIDS

Rama C. Hoetzlein, Graphics Devtech, NVIDIA

## Applications of NNS



Molecular Modeling

Fluid Simulation


Cognitive Science - Behavioral simulation

## Background - What is Fixed-Radius NNS?



Find all neighbors in a fixed radius R
O

O

## Background - What is Fixed-Radius NNS?

Find all neighbors in a fixed radius $R$

- Number of neighbors may vary
- May need to find all fixed-radius neighbors of all particles

O ( $\mathrm{n}^{\wedge} 2$ ) brute force

## Overall Strategy



Spatial Partitioning:

1. Partition space equally into bins
2. Insert each particle into bins
3. Only need to search particles found in neighboring bins

O(Nk)

## Applications



## Applications - Example



Insert Particles


Compute Forces


O(N)
$\mathrm{O}\left(\mathrm{N}\left(\mathrm{k}+\mathrm{D}_{1}\right)\right)$
O( $\left.N\left(k+D_{2}\right)\right)$
$\mathrm{O}\left(\mathrm{N}^{*} \mathrm{D}_{3}\right)$

Nearest Neighbor Search
Domain Specific Cost

## Research Background

Fluid Simulation


2003 Muller
Particle-Based Fluid Simulation for Interactive Apps (SPH)



2006 Randolph Franklin Nearest Point Query on 184,088,599 Points in $\mathrm{E}^{\wedge} 3$ with a Uniform Grid. CPU Grid Search


Fluids v. 3 http: / /fluids3.com
High Performance computing for scientific applications. "Just the basics", Zlib license, Research \& Education, Solves NNS

## Grid Search



Original Input

$$
\begin{array}{llllllllll}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
51 & 70 & 61 & 60 & 71 & 50 & 62 & 61 & 50 & 70
\end{array}
$$

Neighboring Bins


Ideal Layout - Sorted by bins

$$
\begin{array}{llllllllll}
5 & 8 & 0 & 3 & 7 & 2 & 6 & 1 & 9 & 4 \\
50 & 50 & 51 & 60 & 61 & 61 & 62 & 70 & 70 & 71
\end{array}
$$

Neighboring Bins


## Radix Sort

Previous method (Green 2008)

Input

$$
\begin{array}{llllllllll}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
51 & 70 & 61 & 60 & 71 & 50 & 62 & 61 & 50 & 70
\end{array}
$$

Radix Sort
Digit 0

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 51 | 70 | 61 | 60 | 71 | 50 | 62 | 61 | 50 | 70 |
|  | $0=$ | 5 |  | $1=4$ |  | $2=$ | 1 |  |  |

$\left.\begin{array}{|l|l|l|l|l|l|l|l|l|}\hline 70 & 60 & 50 & 50 & 70 & 51 & 61 & 71 & 61\end{array}\right) 62$

Digit 1

$$
\begin{array}{lccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
70 & 60 & 50 & 50 & 70 & 51 & 61 & 71 & 61 & 62 \\
& 5=3 & & 6=4 & & 7=3 &
\end{array}
$$

| 50 | 50 | 51 | 60 | 61 | 61 | 62 | 70 | 70 | 71 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

Repeat step for each digit in key
Output

$$
\begin{array}{llllllllll}
5 & 8 & 0 & 3 & 7 & 2 & 6 & 1 & 9 & 4 \\
50 & 50 & 51 & 60 & 61 & 61 & 62 & 70 & 70 & 71
\end{array}
$$

## Counting Sort mout

| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 51 | 70 | 61 | 60 | 71 | 50 | 62 | 61 | 50 | 70 |



Observations:

1. Goal is to sort by bin
2. Position inside bin in irrelevant
3. Therefore, many duplicate keys
4. Better to perform 1-radix on exact bins, rather than on digits.

## Counting Sort mput

$$
\begin{array}{llllllllll}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
51 & 70 & 61 & 60 & 71 & 50 & 62 & 61 & 50 & 70
\end{array}
$$

| 50 | 51 | 52 |
| :---: | :---: | :---: |
| $0^{5}$ | $\bigcirc^{0}$ |  |
| 60 3 |  | $62$ |
| $\begin{gathered} 70 \quad 1 \\ \bigcirc 9 \end{gathered}$ | $71 \bigcirc^{4}$ | 72 |

Insert + Counts

$$
\begin{aligned}
& \begin{array}{llllllllll}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
51 & 70 & 61 & 60 & 71 & 50 & 62 & 61 & 50 & 70 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 2 & 2
\end{array} \\
& 50=2 \quad 60=1 \quad 70=2 \\
& 51=1 \quad 61=2 \quad 71=1 \\
& 52=0 \quad 62=1 \quad 72=0
\end{aligned}
$$

Prefix Sum

| 50 | 50 | 51 | 60 | 61 | 61 | 62 | 70 | 70 | 71 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 |  | 2 | 3 | 4 |  | 6 | 7 |  | 9 |

Counting Sort


Output

$$
\begin{array}{llllllllll}
5 & 8 & 0 & 3 & 2 & 7 & 6 & 1 & 9 & 4 \\
50 & 50 & 51 & 60 & 61 & 61 & 62 & 70 & 70 & 71
\end{array}
$$

## What is a good Grid Cell Size?



Too many grid cells to check
e.g.
$5 \times 5 \times 5=125$ cells

$\mathrm{h}=\mathrm{d} / 2.00>3$

$h=d / 3.00$

## Algorithm Comparison

CUDA Particles (Radix Sort)

Insert Particles
assign particle to cell

## Sort Particles

thrust:sort_by_key example: CUDA RadixSort for 1 to 4 (each byte in key) Bin Counts
Bin Prefix Sum
RadixAddOffsetAndShuffle

Reindex (copy particles in order)
Time integration

Fluids v. 3 (Counting Sort)

Insert Particles
assign particle to cell

Sort Particles

Bin Sums
Bin Prefix Sum
Radix Offset and Shuffle

Relndex (copy particles in order)
Time integration

## Algorithm Comparison

CUDA Particles (Radix Sort)
Fluids v. 3 (Counting Sort)

1 Insert Particles
assign particle to cell

Sort Particles
thrust:sort_by_key example: CUDA RadixSort for 1 to 4 (each byte in key) Bin Counts
Bin Prefix Sum
RadixAddOffsetAndShuffle

1 Reindex (copy particles in order)
1 Time integration
15 Kernel calls / Frame

## Bin Counting



How many? 2 3

1

## Bin Counting



Old Method
Parallel Sums


Each GPU thread computes binary sets of sums.

## Bin Counting



New Method
Atomic
Adds

| +1 | +1 | +1 | +1 |
| :---: | :---: | :---: | :---: |
| +1 | +1 |  | +1 |
|  | +1 |  |  |
| 2 | 3 | 1 | 2 |

Each particle atomic-adds into bin at the same time bin is determined.

## Results - Queries per Second



## Results - Time for Each Step



## Results - Fluids Example

* Results do not show fluid accuracy or time step.




$$
\begin{gathered}
\text { Fluids v.2, } 2009 \\
16,384 \text { at } 32 \mathrm{fps}
\end{gathered}
$$

same hardware 11x faster Fluids v. 3, 2013 193,487 at 32 fps

## Thank you!

- Rama C. Hoetzlein, rama@rchoetzlein.com
http://fluids3.com

